17 Oscillations

17.1 Simple harmonic oscillations

Candidates should be able to:

1 understand and use the terms displacement, amplitude, period, frequency, angular frequency and phase difference in the context of oscillations, and express the period in terms of both frequency and angular frequency

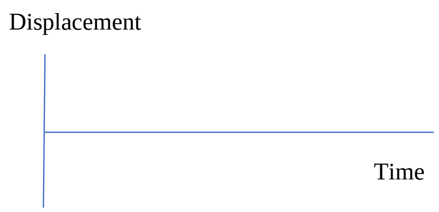
2 understand that simple harmonic motion occurs when acceleration is proportional to displacement from a fixed point and in the opposite direction

3 use $a = -\omega^2 x$ and recall and use, as a solution to this equation, $x = x_0 \sin \omega t$

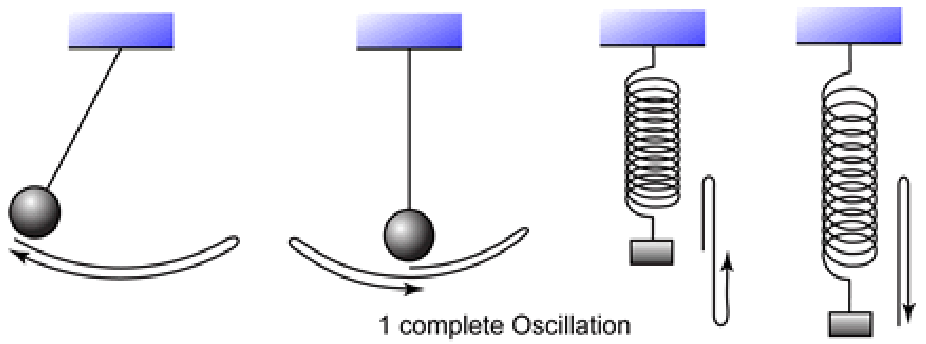
4 use the equations $v = v_0 \cos \omega t$ and $v = \pm\omega\sqrt{(x_0^2 - x^2)}$

5 analyse and interpret graphical representations of the variations of displacement, velocity and acceleration for simple harmonic motion

- An oscillation is defined as **repeated back and forth movements on either side of any equilibrium position.**
- When the object stops oscillating it returns to its equilibrium position
- An oscillation is a more specific term for a vibration
- An oscillator is a device that works on the principles of oscillations
- Oscillating systems can be represented by displacement-time graphics
- The motion in the graphs is described as sinusoidal



- Some properties of oscillation that candidates must be familiar with include
  -**Displacement (x)** of an oscillating system is defined as
  The distance of an oscillator from its equilibrium position
  -**Amplitude (x_o)** is defined as
  The maximum displacement of an oscillator from its equilibrium position
  -**Frequency (f)** is defined as
  The number of complete oscillations per unit time (f = 1/T)
  -**Time period (T)** defined as
  The time taken for one complete oscillation, in seconds



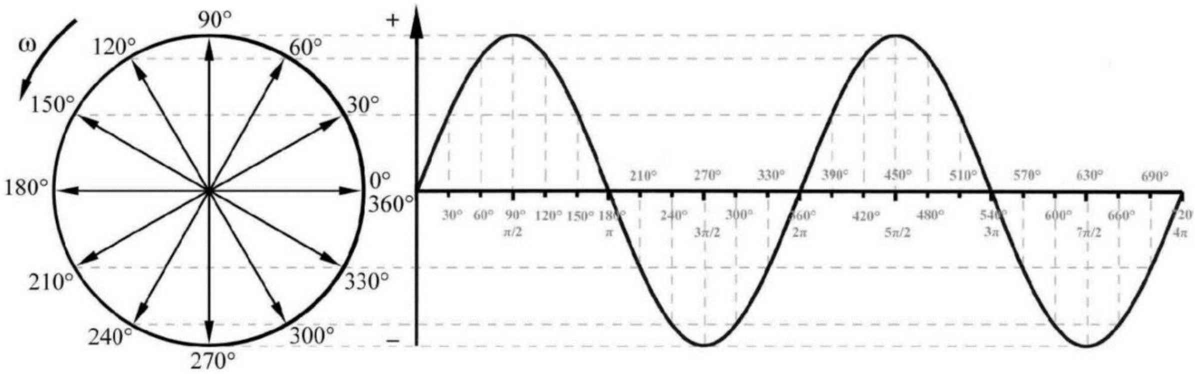
-Angular frequency (**ω**) is defined

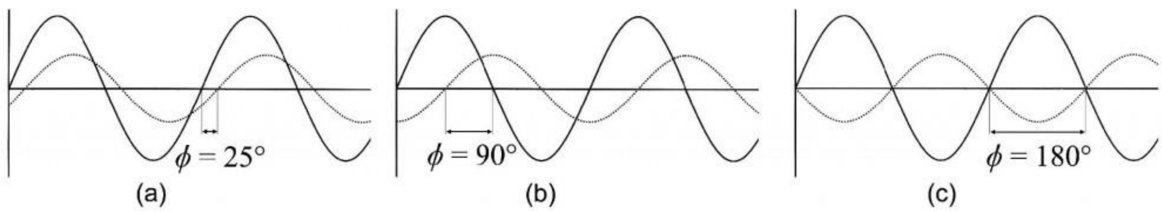
The rate of change of angular displacement with respect to time

This is a scalar quantity measured in rad s^{-1} defined by the equation

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Phase difference is how much one oscillator is in front or behind another





-When the relative position of two oscillators is equal, they are in phase

-When one oscillator is exactly half a cycle behind another, they are said to be in anti-phase

-Phase difference is normally measured in radians or fractions of a cycle

-When two oscillators are in antiphase, they have a phase difference of **π radians**

- Simple harmonic motion (SHM) is **a type of oscillation in which the acceleration of a body is proportional to its displacement, but acts in the opposite direction.**
- E.g. of SHM oscillators are

  -pendulum of a clock

  -mass of a spring

- Acceleration a and displacement x can be represented by the defining equation of SHM:

$$a \propto -x$$

- An object in SHM will have a restoring force to return it to its equilibrium position
- This restoring force is directly proportional, but in the opposite direction, to the displacement of the object (F=ma).
- The acceleration of an object oscillating in SHM is:

$$a = -\omega^2 x$$

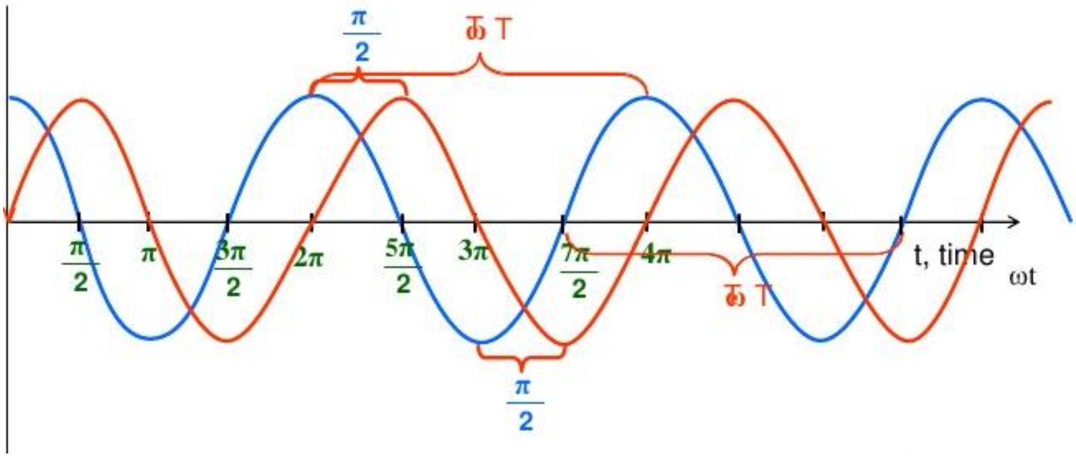
- based on the equation above, acceleration is maximum when x is at maximum (x_0 or amplitude)
- The negative sign shows that when the object is displaced to the left, the acceleration is to the right and vice versa.
- An equation for SHM displacement is

$$x = x_0 \sin(\omega t)$$

- This equation is useful for finding the position of an object in SHM if you know the angular frequency (ω) and time (t).
- Another possible equation for SHM is

$$x = x_0 \cos(\omega t)$$

- The graph below shows both the sin (red) and cos (blue) solutions:



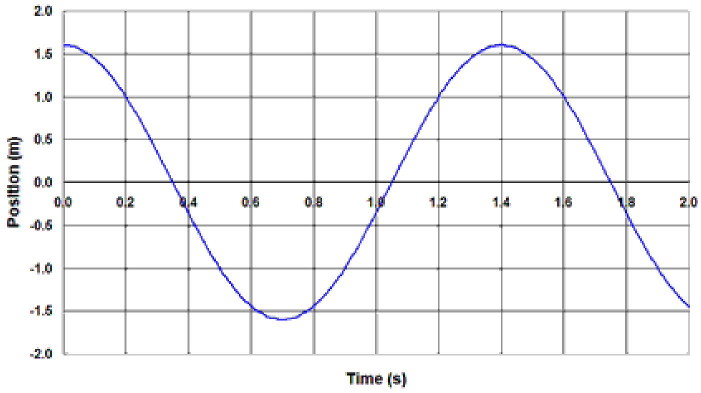
- The speed of an oscillator in SHM can be determined with the following equation

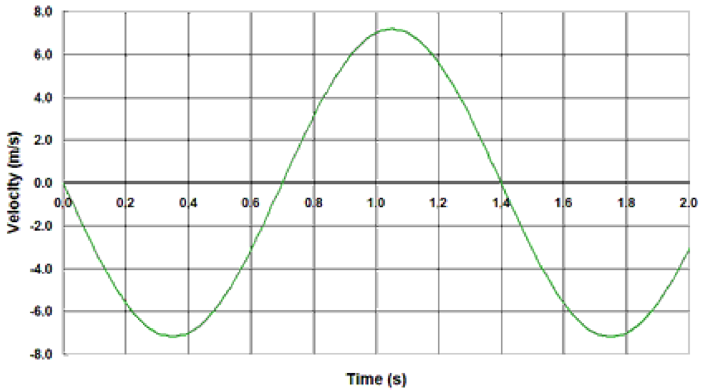
$$v = v_o \cos(\omega t)$$

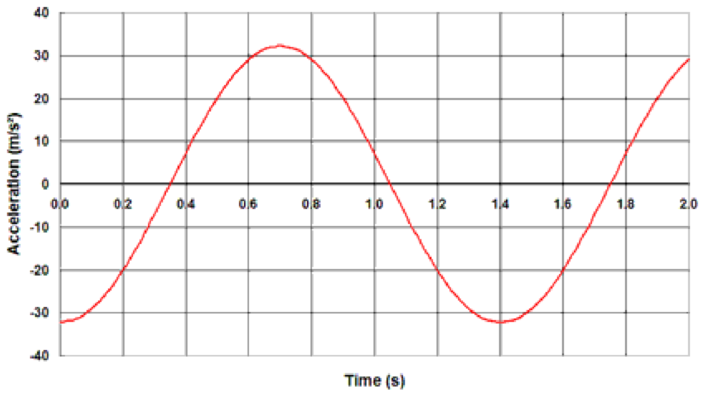
- Here v is the speed (ms^{-1}) and v_o the maximum speed.
- The equation above tells us that speed is maximum when ω = 0,180,360 etc. since cos (0) = 1
- To find how the speed changes with the oscillator's displacement (instead of ω) you can use the following equation

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

- The following graphs below show the displacement, velocity and acceleration graphs of an object in SHM.
- Velocity of an oscillator can be determined from **the gradient of the displacement-time graph**: dx/dt.
- For the velocity graph, velocity is at its maximum when the displacement is zero.
- Acceleration can be determined from **the gradient of the velocity-time graph**: dv/dt.
- The maximum value of the acceleration is when the oscillator is at is maximum displacement.







17.2 Energy in simple harmonic motion

Candidates should be able to:

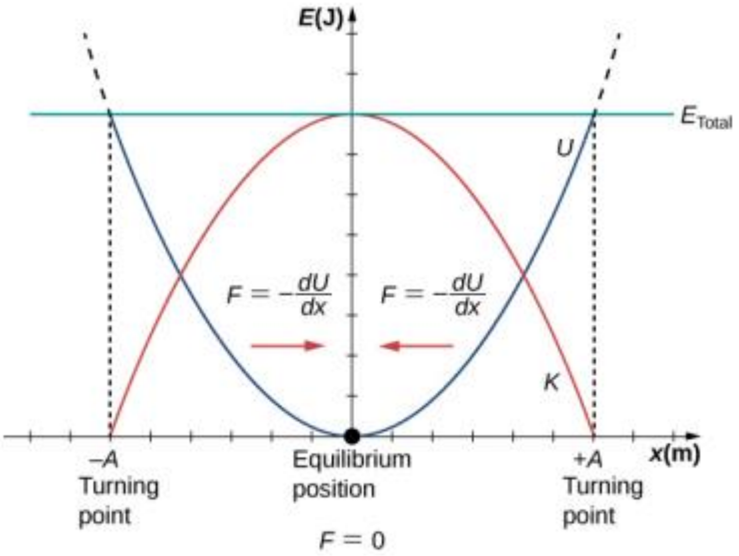
1 describe the interchange between kinetic and potential energy during simple harmonic motion

2 recall and use $E = \frac{1}{2}m\omega^2 x_0^2$ for the total energy of a system undergoing simple harmonic motion

- During SHM, energy is constant exchanged between KE and PE.
- When one goes up, the other goes down and vice versa.
- E.g., the PE of a pendulum swing is maximum when it is at the top of the swing whereby it momentarily stops (KE=0) and reverse direction.
- The KE is maximum at the point of equilibrium (bottom PE=0)
- Speed (v) is max when displacement x = 0. Hence KE is maximum

- At max displacement $x = x_0$ (amplitude), PE = max while KE = 0
- SHM is therefore converting between PE and KE all the time.
- The total energy of the system

Total energy = KE + PE



The total energy of a system undergoing SHM is defined by

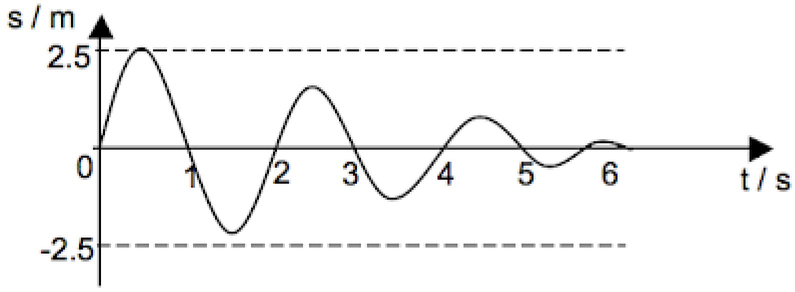
$$E = \frac{1}{2} m \omega^2 x_0^2$$

17.3 Damped and forced oscillations, resonance

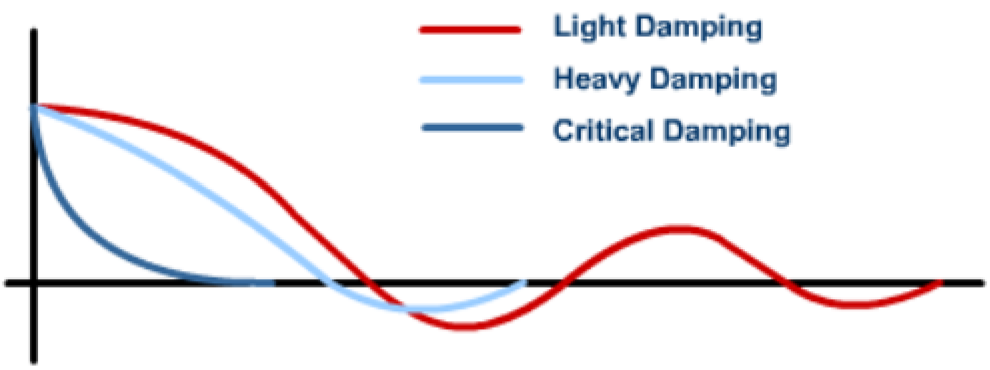
Candidates should be able to:	
1	understand that a resistive force acting on an oscillating system causes damping
2	understand and use the terms light, critical and heavy damping and sketch displacement–time graphs illustrating these types of damping
3	understand that resonance involves a maximum amplitude of oscillations and that this occurs when an oscillating system is forced to oscillate at its natural frequency

- All oscillations eventually come to a stop due to resistive forces, such as friction or air resistance (drag).
- These resistive forces act on an oscillating system causing **damping**.
- **Damping** is defined as **the reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system**.
- Damping continues until the oscillator comes to rest at the equilibrium position.

- **Frequency** does not change during damping only the **amplitude** of the oscillation decreases.

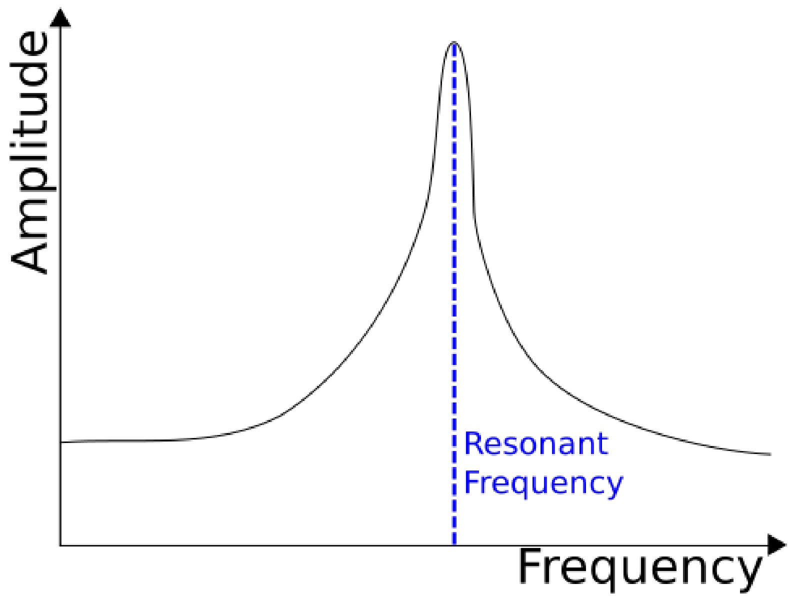


- There are three types of damping
  -Light damping: amplitude decays exponentially with time. E.g., pendulum swinging with decrease amplitude until it stops.
  -Critical damping: The oscillator will return to rest in the shortest time possible **without oscillation**. E.g., car suspension system.
  -Heavy damping: Takes a long time to return to rest **without oscillation.** E.g., door dampers to prevent sudden shut.



- **Forced oscillations** are defined as **periodic forces which are applied in order to sustain oscillations**.
- Without forced oscillations, a damped system will eventually come to rest.
- The frequency of forced oscillations is called the **driving frequency (f)**.
- All systems have a **natural frequency (f_0)**.
- The **natural frequency (f_0)** is **the frequency of an oscillation when the oscillating system is allowed to oscillate freely**.
- When the **driving frequency (f)** matches the **natural frequency (f_0), resonance** is achieved.
- When resonance occurs, the driving frequency applied to an oscillating system is equal to its natural frequency, the amplitude of the oscillation will increase.
- At resonance energy transferred from the driver to the system is at its most efficient point.

- A resonance curve is a graph of driving frequency f against amplitude of oscillations.



- In a resonance curve as shown above,

  -When $f<f_0$ the amplitude of oscillations increases

  -At the peak where $f = f_0$ the amplitude is at its maximum. This is resonance.

  -When $f> f_0$ the amplitude of oscillations starts to decrease.