19 Capacitance

19.1 Capacitors and capacitance

Candidates should be able to:

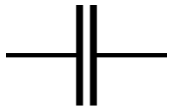
1 define capacitance, as applied to both isolated spherical conductors and to parallel plate capacitors

2 recall and use $C = Q / V$

3 derive, using $C = Q / V$, formulae for the combined capacitance of capacitors in series and in parallel

4 use the capacitance formulae for capacitors in series and in parallel

- Capacitors are electrical devices used to store energy in electronic circuits.
- The circuit symbol for capacitor is shown below

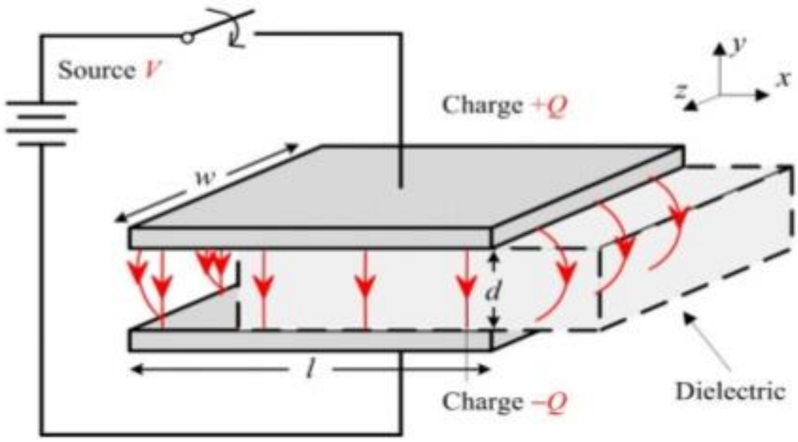


- They come in two forms

  -Isolated spherical conductor

  -Parallel plates

- The unit of capacitor is **capacitance**.
- **Capacitance** is defined as **the charge stored per unit potential difference**.
- The higher the capacitance, the greater the energy that can be stored in a capacitor.
- A parallel plate capacitor is made up of two conductive metal plates connected to a voltage supply



- The negative terminal of the voltage supply pushes electrons onto one plate, making it negatively charged.
- The electrons are repelled from the opposite plate, making it positively charged.

- There is a commonly a dielectric in between the plates to prevent the charge does not free flow between them.
- The capacitance (C) of a capacitor is defined by the equation

$$C = \frac{Q}{V}$$

- The SI unit is in **Farad (F)**
- If the capacitor is made of parallel plates, Q is the charge on the plates and V is the potential difference across the capacitor.
- For spherical conductor, Q is the charged stored on its plates.
- The capacitance of a charged sphere is defined by the charge per unit potential at the surface of the sphere.
- Recall that the potential (V) of an isolate point charge is given by

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Substituting into the capacitance equation we get the equation for capacitance (C) of a sphere

$$C = 4\pi\varepsilon_0 r$$

- For capacitor in **series**, recall that the total voltage (V_T) is given by

$$V_T = V_1 + V_2$$

Substituting

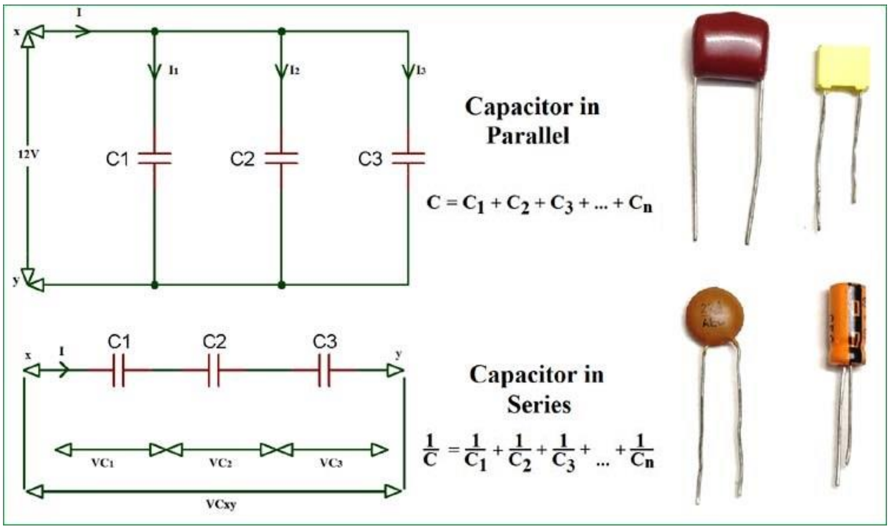
$$V = \frac{Q}{C}$$

Into the equation above we get

$$\frac{Q}{c_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Since the current is the same for a series circuit, Q will cancel out. If you have more capacitors the equation will become

$$\frac{Q}{c_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_2} + \cdots.$$



- For capacitors in parallel start with

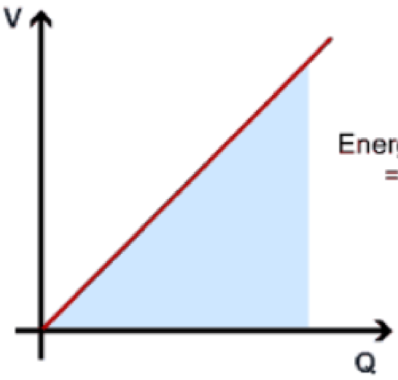
$$Q_T = Q_1 + Q_2$$

You should get

$$C_T = C_1 + C_2 + C_3 + \ldots$$

19.2 Energy stored in a capacitor

Candidates should be able to:	
1	determine the electric potential energy stored in a capacitor from the area under the potential–charge graph
2	recall and use $W = \frac{1}{2}QV = \frac{1}{2}CV^2$



- The charge (Q) on a capacitor is **directly proportional** to its potential difference (V).
- The area under the curve of a potential-charge graph is equal to the area under a **triangle**.
- This **area** is the **energy stored** in a capacitor.
- The energy stored (W) is therefore

$$W = 1/2\ QV$$

Substituting Q = CV we get

$$W = \frac{1}{2} CV^2$$

19.3 Discharging a capacitor

Candidates should be able to:	
1	analyse graphs of the variation with time of potential difference, charge and current for a capacitor discharging through a resistor
2	recall and use $\tau = RC$ for the time constant for a capacitor discharging through a resistor
3	use equations of the form $x = x_0 e^{-(t/RC)}$ where x could represent current, charge or potential difference for a capacitor discharging through a resistor

- When a capacitor is being charged, the electrons flow from the positive to negative plate.
- When the capacitor is being discharged through a resistor, the electrons flow back from negative plate to the positive plate until there are equal number of electrons on each plate.
- At the start of the discharge, the current is large and gradually falls to zero.

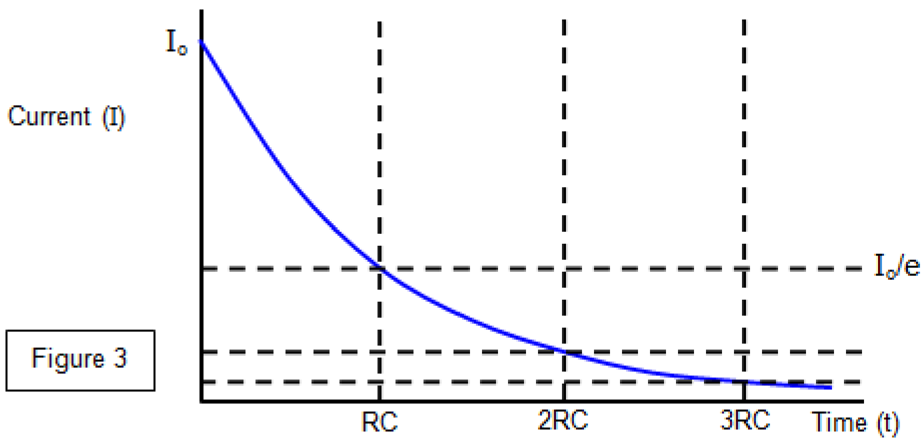


Figure 3

- As a capacitor discharges, the I, V and Q all decrease exponentially.
- This is represented by an exponential decay in the graph above.
- V and Q versus time graphs have a similar shape as well.
- The rate at which a capacitor discharges depends on the **resistance** (R) of the circuit.
- A **high resistance** will **slow down the discharge** since the current will decrease.
- A **low resistance** will **increase the rate of discharge** since current can flow more freely.
- The time constant of a capacitor discharging through a resistor is a measure of how long it takes for the capacitor to discharge.
- **Time constant (τ)** is defined as **the time taken for the charge of a capacitor to decrease to 0.37 of its original value**

$$\tau = RC$$

- The equations below can be used to determine how much current (I), potential difference (V) and charge (Q) left after a certain amount of time from its initial I_0, V_0 and Q_0.

$$I = I_0 e^{-\frac{t}{RC}}$$
$$V = V_0 e^{-\frac{t}{RC}}$$
$$Q = Q_0 e^{-\frac{t}{RC}}$$