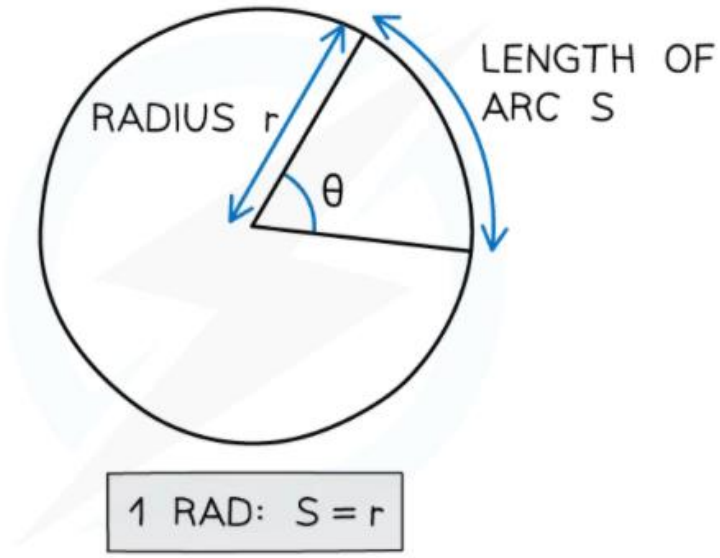
12 Motion in a circle

12.1 Kinematics of uniform circular motion

Candidates should be able to:	
1	define the radian and express angular displacement in radians
2	understand and use the concept of angular speed
3	recall and use $\omega = 2\pi / T$ and $v = r\omega$

- The **angular displacement** of a body is the change in angle (radians, degree or revolutions) through which the body rotates around a circle
- **Angular displacement** is the ratio of:

$$\Delta\theta = \frac{\Delta s}{r}$$



- A **radian** (rad) is defined as **the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle**
- Radians is usually written in term of π
- For a rotation for a complete circle (360^0), the radians is 2π ($\Delta\theta = 2\pi r/r$)
- For a rotation for half a circle (180^0), the radians is π ($\Delta\theta = \pi r/r$)
- To convert degrees to radian use

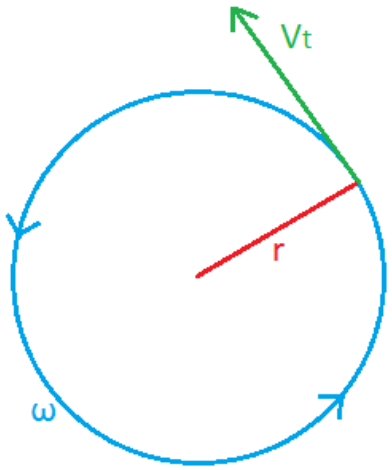
$$\frac{\theta^0}{180^0} \times \pi = \theta rad$$

- Angular speed (ω) is defined as the **rate of change in angular displacement with respect to time** the unit is measured in **rad s^{-1}** (or **angle s^{-1}** or **rev s^{-1}**)

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

- The **tangential velocity** is the velocity measured **at any point tangent to a rotating body**
- The SI unit for tangential velocity is **ms^{-1}**
- The equation is given as

$$V_t = r\omega$$

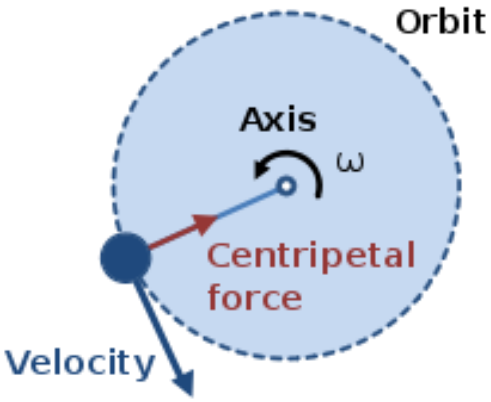


- Use **rad s^{-1}** for ω!
- The further the object is from the **centre of the circle (r)**, the greater the velocity needed to complete a full circle
- Eg. track running

12.2 Centripetal acceleration

Candidates should be able to:	
1	understand that a force of constant magnitude that is always perpendicular to the direction of motion causes centripetal acceleration
2	understand that centripetal acceleration causes circular motion with a constant angular speed
3	recall and use $a = r\omega^2$ and $a = v^2/r$
4	recall and use $F = mr\omega^2$ and $F = mv^2/r$

- During a uniform circular motion, an object is **continuously changing direction**.
- Since **velocity** is a **vector**, the **change in direction** would imply that there is an **acceleration** on the object.
- This acceleration is called **centripetal acceleration**.
- The centripetal acceleration is caused by **centripetal force**.
- Centripetal force means **centre seeking force** as it always acts towards the centre.
- Note that **speed** is **constant** even if velocity is changing.
- This is because **speed** is a **scalar**.
- **Angular speed (ω)** stays **constant** as well.



Centripetal acceleration (a_c) is given by

$$a_c = \frac{v_t^2}{r}$$

Applying

$$V_t = r\omega$$

You get

$$a_c = r\omega^2$$

Centripetal force (F_c) can therefore be calculated using

$$F_c = ma_c$$

Which will give you

$$F_c = \frac{mv_t^2}{r}$$

Or

$$F_c = mr\omega^2$$